Density of states, screening and the width of the quantum Hall plateaus

K. Tsemekhman, V. Tsemekhman

Department of Physics, University of Washington, Box 351560, Seattle, Washington 98195

C. Wexler

Department of Physics, Box 118440, University of Florida, Gainesville, Florida 32611

(7 Oct 1998)

We present a consistent treatment of the quantum Hall effect within the electrostatic approximation. We derive the form of the density of states (DOS) which differs from the usual gaussian shape valid for non-interacting electrons. Below a crossover temperature, this DOS gradually transforms into the non-interacting DOS near the Fermi energy. We obtain an estimate of this crossover temperature, which determines the range of validity of this framework, and allows a reconciliation of the width and temperature dependence of the quantum Hall plateaus with experiments. The DOS dramatically enhances the electron-lattice relaxation rate, thus reducing the possibility of electron overheating.

PACS numbers: 73.40.Hm,73.50.-h, 73.50.Jt

I. INTRODUCTION

The quantum Hall effect (QHE) is usually thought of as a manifestation of the absence of scattering of electrons carrying the Hall current. This is caused by the Fermi level falling between two mobility edges. Each mobility edge in an infinite disordered system is the energy of a unique extended state on a given Landau level (LL). It has been shown, numerically and in a number of approximations[1–3], that the density of states (DOS) of non-interacting electrons is symmetric around the extended state energy of each LL, and has a width that depends on the strength of disorder but which is always much smaller than the cyclotron energy $\hbar\omega_c$, with $\omega_c = eB/mc$. This density of states proved to be consistent with the qualitative explanation of the widths of QH plateaus[4] and with the numerical values of critical exponents describing the scaling at phase transitions between QH plateaus[3]. Interaction between electrons has, however, been left out of those treatments, without a strong *a priori* justification.

On the other hand, Efros[5] has suggested that, in presence of long-range fluctuations of the disorder potential, interactions between electrons are crucial, leading to collective screening of the disorder potential. Efros showed that, if the electrostatic approximation is appropriate (which is true when the screening radius is much smaller than any other relevant length), there could be two types of screening which he denoted "linear" and "non-linear". Linear screening is essentially metallic-like: in a region with linear screening the energies of two-dimensional electrons are confined to an interval of the order of temperature around the chemical potential and states on the top LL are partially filled. On the contrary, non-linear screening is much less effective. In such regions the amplitude of fluctuations of the disorder potential is reduced due to electric charge of electrons, but this reduction is limited by the maximum density of elec-

trons on the LL and by the inter LL energy gap.

The existence of linear screening is supported by the theory of compressible edge strips in a gate-confined two-dimensional electron system (2DES)[6]. Although the nature of the potential being screened in these long compressible strips is quite different from that of the disorder potential, the difference in the description of screening in these two cases is relatively small. Conditions for the validity of the electrostatic approximation itself, on the other hand, should be universal. If such conditions are satisfied, one can expect a considerable fraction of the area of the 2DES to be covered by compressible regions and, therefore, a large density of states near the Fermi level (and at energies differing by $\hbar\omega_c$). Such DOS (we will refer to it as the *screened* DOS) is completely different from the corresponding DOS for non-interacting electrons. The difference leads to appreciable differences in the predicted widths of QH plateaus[5], apparently, for the rates of various scattering processes. Similar considerations were also successfully applied in the past to a quantitatively correct theory of the current breakdown of the QHE[7].

Dissipative transport properties at sufficiently high temperatures are due to thermal activation of electrons to unfilled LL (or due to holes in filled LL). It was shown both experimentally[8,9] and in a number of theoretical models[10–12] that the average dissipative conductivity of the 2DES in the quantum Hall regime satisfies the equation

$$\sigma_{xx} = \sigma_0 \exp(-\epsilon_a/kT_e) , \qquad (1)$$

where σ_0 is generally a non-universal constant ranging between $0.5e^2/h$ and $5e^2/h$, ϵ_a is the activation energy defined as the distance between the Fermi level and the energy of the *extended* state, and T_e is the temperature of electron gas. We keep the subscript in T_e to distinguish it from the lattice temperature T_b in cases when these are different. The range of validity of Eq. (1) will

be discussed later in this paper. We want to stress at this point, however, that Eq. (1) implies an *exponential* smallness of σ_{xx}, and therefore, exponentially small Joule heating due to scattering inside the system.

One of the consequences of the non-interacting DOS is the suppression of scattering events by delocalized states since they are all deep under the Fermi level. Therefore, the energy relaxation rate via phonon emission would also be small in this case. If this non-interacting approximation was valid, under some conditions the phonon-mediated energy relaxation could become slower that the Joule heating, resulting in observable overheating of the 2DES[8,9,13,14]. Alternatively, the interacting DOS provides for a macroscopic fraction of states at the Fermi level and, therefore, allows a very effective coupling of the 2DES to the lattice. Overheating, in this case, becomes negligible.

In this paper we address the question of the range of validity of both non-interacting DOS and screened DOS. The former is valid at low temperatures, while the latter dominates in the higher temperature regimes. We present an estimate of the crossover temperature falling in the range between 10 mK and 1K depending on various parameters of the sample and on experimental quantities. At temperatures above this crossover value, the screened DOS leads to a gigantic enhancement of the typical phonon emission rates and, therefore, to the absence of overheating in the QH regime.

The paper is organized as follows. In Sec. II we discuss the screening of a long-range disorder potential, closely following the formulation by Efros[5]. The concept of screening is then used to introduce the new form of the DOS of interacting electrons. In Sec. III we examine the range of validity of the electrostatic approximation and present an intuitive picture of the physics beyond this range. Section IV deals with the evolution of the quantum Hall effect with temperature. We derive an estimate for the crossover temperature below which the DOS behaves more closely to the conventional DOS for non-interacting electrons in a random potential, at least close to the Fermi energy. We also propose a theory of the melting of the QH liquid and present numerical results supporting our understanding of the development of the DOS with the temperature. Finally, in Sec. V we discuss dissipative processes in the context of the screened DOS, which dramatically enhances the electron-lattice relaxation rate thus eliminating electron overheating in the QH effect.

II. SCREENING

We will describe the QH system in a hierarchy of approximations. In each hierarchical level the common goal will be the determination of the spatial distribution of filling factors for each LL [$n^{(N)}(\mathbf{r})$]. In a strong magnetic, for sufficiently slowly varying external potentials (of any nature) it is a good approximation to consider all the states on consecutive LLs to be locally separated by $\hbar\omega_c$.

The hierarchy starts with a simple electrostatic approximation. A local filling factor is used to obtain the classical charge density distribution $\rho(\mathbf{r})$ using

$$\rho(\mathbf{r}) = \frac{1}{2\pi l^2} \sum_N n^{(N)}(\mathbf{r}) \,, \qquad (2)$$

where $l = (\hbar c/eB)^{1/2} \sim 100$ Å is the magnetic length. This approximation is equivalent to finding the electron density distribution $\rho(\mathbf{r})$ which minimizes the free energy of the system. At zero temperature this corresponds to the minimization of a Hamiltonian consisting of the potential energy of interactions between electrons, interaction with the background disorder and quenched kinetic energy:

$$\mathcal{H}_1 = \frac{e^2}{2\epsilon} \iint d^2\mathbf{r}\, d^2\mathbf{r}' \frac{\rho(\mathbf{r})\rho(\mathbf{r}')}{|\,\mathbf{r} - \mathbf{r}'\,|} + e \int d^2\mathbf{r}\, \rho(\mathbf{r})V(\mathbf{r})$$
$$+ \sum_N (N + \frac{1}{2})\hbar\omega_c \int d^2\mathbf{r}\, n^{(N)}(\mathbf{r}) \,. \qquad (3)$$

The third term in the right-hand side of Eq. (3) reflects the existence of a gap between the Landau levels. The problem corresponding to Hamiltonian $\mathcal{H}_1$ was first analyzed and then solved numerically by Efros and co-workers[5,15,16] for a disorder potential created by remote donors in a plane parallel to the 2DES. As these authors pointed out, for essentially any disorder potential the response of the electron system results in two types of screening which they labeled *linear* and *non-linear*. In the linear screening regions the total electrostatic potential

$$\Phi(\mathbf{r}) = \frac{e}{\epsilon} \int d^2\mathbf{r}' \frac{\rho(\mathbf{r}')}{|\,\mathbf{r} - \mathbf{r}'\,|} + eV(\mathbf{r}) \qquad (4)$$

is flat and satisfies the equation

$$\Phi(\mathbf{r}) = \mu - (N + \frac{1}{2})\hbar\omega_c \,, \qquad (5)$$

where μ is the chemical potential and N is the top partially-filled LL in the vicinity of point $\mathbf{r}$. The filling factor $n^{(N)}(\mathbf{r})$ in such areas, which are called *compressible*, varies between zero and one.

In contrast to this situation, there are also *incompressible* regions of non-linear screening which are characterized by a complete filling of the top-most Landau level $n^{(N)}(\mathbf{r}) = 1$. In these cases the potential satisfies the inequalities

$$\mu - (N + \frac{3}{2})\hbar\omega_c \;<\; \Phi(\mathbf{r}) \;<\; \mu - (N + \frac{1}{2})\hbar\omega_c \,. \qquad (6)$$

Before going up in hierarchy, we give our interpretation of these equations. With respect to the effectiveness of screening, one can imagine two situations: a compressible

region percolates or an incompressible region percolates. In the former situation, there are extended states at the Fermi energy and the system behaves essentially as a metal. The latter scenario describes the quantum Hall effect since in this case, the extended states are away from the Fermi level, scattering processes are suppressed and the Hall conductance is quantized with a very high accuracy.

A typical picture of an inhomogeneously filled 2DES is shown on Fig. 1. It is the result of our numerical simulations of the 2DES at finite temperature, the essence of which will be described in the following Sections. In the QH regime (Fig. 1(a)) the majority of the plane is taken by incompressible (completely filled LLs) areas. Yet, compressible lakes still take a finite fraction of the sample. By the virtue of Eq. (5) we conclude that a finite fraction of the total number of states have energy equal to the chemical potential. Since all the partially filled regions have sizes smaller than the sample size, the corresponding quantum states ought to be localized and do not contribute to dissipation. To a limited extent, the situation is similar to a common 3-dimensional insulator system, where there are states present at the Fermi level but all of them are localized in space. However, there is a major departure from this picture: while in a common insulator the density of states is smooth around the Fermi level (or even vanishes in semiconductors due to the Coulomb gap), in the case under study it is strongly peaked around the Fermi level. Finite temperature and neglected correlations, the effects which we consider below, tend to smear the peak, but the weight of partially filled states remains finite.

Quite remarkably, the form of the DOS does not change dramatically with the onset of the dissipative regime with a percolating compressible region [see 1(a),(b) and Figs 2(a),(b)]. While the landscapes of filling factor and potential do undergo major changes with the density of electrons passing through the critical value corresponding to the boundary between the two percolating regimes, the DOS changes only quantitatively and in a smooth fashion. The existence, in both regimes, of a finite fraction of states on the Fermi level, allows a percolation transition to occur without a visible transformation of the DOS. During the transition, most of the states in the *percolating* compressible region remain localized. The presence of these localized states *at the Fermi level*, however, does have profound effects since these states can actively participate in low energy scattering processes, giving rise to finite dissipative conductivity.

Comparing the situation above to the metal-insulator transition in 3-dimensional systems, we note that the latter one occurs when the states on the Fermi level transform from localized to extended: the number of such states in the localized regime is so small that they are always spatially separated from each other by wide potential barriers and can only conduct via exponentially small hopping. One can say that in conventional 3D metal-insulator transition the Fermi level moves with respect
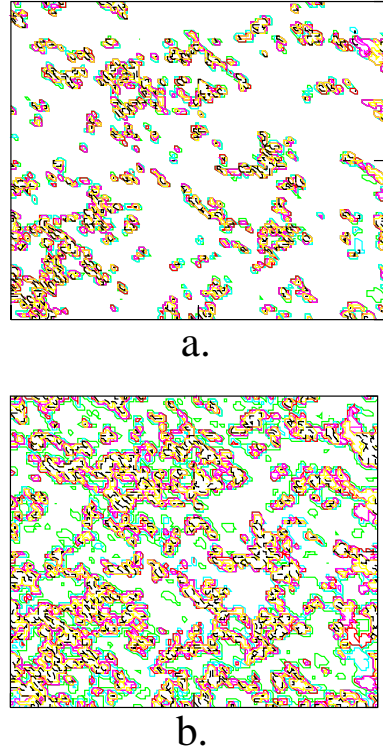

a.

b.

FIG. 1. Electron density distribution in the 2DES; shaded areas represent compressible, partially filled states on the first or second LL, white regions are incompressible. a.: Incompressible region percolates (QH regime); b.: Compressible region percolates.

to the DOS; in the 2D system under consideration both the Fermi energy and the DOS are almost rigid. Instead, the main transformation is the motion of the energy of the extended state, which approaches the Fermi energy as the system moves *away* from the QH regime.

We conclude that the structure of the DOS near the Fermi level, in a regime when the electrostatic approximation is justified, plays a crucial role in the mechanisms of conductivity. The origin of this DOS can easily be seen from the Hamiltonian of Eq. 3. Compared to a typical 3-dimensional electron system, its kinetic energy term is quenched by the strong magnetic field and replaced by the quantized energy of electrons on a given LL. As long as this replacement is valid and the number of filled Landau levels is sufficiently small, there are no further degrees of freedom for the electron cloud other than the potential energy. In 3D systems the states with the same average potential energy (in the same spatial region) would have different average kinetic energies thus providing for a continuous spectrum of the energies of these quantum states; *no* preference in the DOS would then be given to the Fermi energy. In the 2DES in a strong magnetic field, the potential energy is minimized and all the states in the "screened" region tend to have the same *total energy*, equal to the Fermi energy.
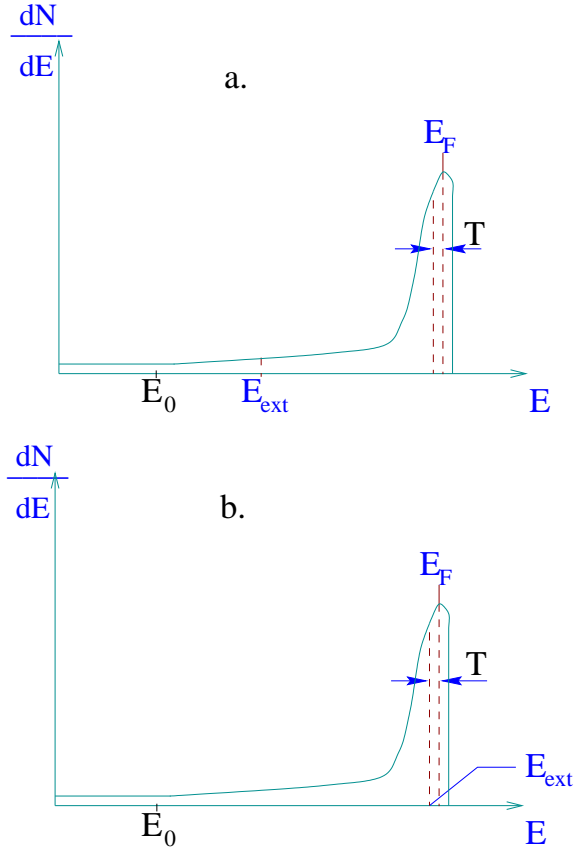
FIG. 2. Density of states for interacting electrons at two different values of the magnetic field. a.: In the quantum Hall regime; b.: In transitional regime. E_0 is the center of LL in the corresponding non-interacting system. Note that the only visible difference is the position of the energy of the extended states relative to the Fermi level. The DOS is always peaked at the Fermi level.

III. RANGE OF VALIDITY OF THE ELECTROSTATIC APPROXIMATION

Let us analyze more carefully the assumptions of the electrostatic approximation. We begin by estimating the size of the neglected kinetic energy term. Consider a mean-field equation for a single electron in the presence of the random potential $V(\mathbf{r})$. Under the assumption of a sufficiently weak and slow varying electric field $\mathcal{E} \ll \hbar\omega_c/el$ one can expand locally $V(\mathbf{r})$ and write the corresponding one-particle Hamiltonian keeping only the linear term:

$$H = \frac{(\mathbf{p} + e\mathbf{A}/c)^2}{2m} - e\,\mathcal{E}\,y \;, \tag{7}$$

where $\mathcal{E} \sim u_0/e\lambda$ is a typical local electric field due to a disorder potential of size u_0 and correlation length λ. It is natural to choose the direction perpendicular to the local electric field as a quasi-translationally invariant one. Then, within the region where this approximation is valid, it is possible to enumerate the solutions by the wave-vectors corresponding to the quasi-translationally invariant direction, as it is done in a common Landau equation[17].

This electric field produces a quadratic energy shift which can be referred to as the kinetic energy, since it is quadratic in the drift velocity $\mathcal{E}/B$. A simple estimate gives the value $(e\mathcal{E}l)^2/\hbar\omega_c \sim 0.01\,\hbar\omega_c$ for long wavelength disorder potential. This justifies neglecting the kinetic energy term in the system we are interested in, since fluctuations of the bare potential commonly exceed $\hbar\omega_c$. Moreover, in screened regions, the minimization of the potential energy leads to a simultaneous reduction of the kinetic term as well (decreasing of the potential fluctuation of a certain wavelength implies a decrease of the corresponding electric field by the same factor). Therefore, inclusion of the quadratic kinetic energy term does not produce any major change, which validates the electrostatic approximation used in Sec. II.

The electrostatic picture can still be changed by quantum effects. Quantum correlations contribute an additional term into the energy density of the homogeneous 2DES[15,18]. One possible form of writing this term is[15]:

$$H_0(\nu) = \sqrt{2\pi}\,\frac{e^2}{\epsilon}\,n_0^{3/2}\,g(\nu) \;, \tag{8}$$

where n_0 is the density on each Landau level, ν is the filling factor and $g(\nu)$ is some complicated function of ν of order of unity at $\nu = 1$. This term is responsible for a finite screening radius r_s, contrary to the classical value $r_s = 0$. It is clear that there are only two characteristic length scales where quantum effects set in: the size of the wavefunction and the average distance between particles on each Landau level. The first length is entirely determined by the magnitude of magnetic field while the second one depends also on the density. When the filling factor is close to $\nu = 1$ both length scales are of the order of magnetic length; the screening length should, therefore, be of the order of the magnetic length as well. This agrees with an estimate for $q_s = 2\pi/r_s$ from Eq. (8):

$$q_s = 2\pi\frac{e^2}{\epsilon}\left(\frac{d^2 H_N}{dn^2}\right)^{-1} \;. \tag{9}$$

Although Eq. (8) may not be a good approximation for such an inhomogeneous system as the one we study, it gives a good order-of-magnitude estimate, at least for external potentials with wavelengths $\lambda > l$. The meaning of q_s here is the same as in the usual 3-dimensional theory of screening: it measures how effectively a given harmonic V_q of an external potential is reduced due to electron interactions. In the 2DES with Coulomb interaction in all the Fourier components of external potential one has to make a replacement $q \rightarrow q + q_s$ in order to reproduce the corresponding harmonic of the screened potential. The Fourier component of the screened potential is $\tilde{F}_{\mathbf{q}} \sim (q + q_s)^{-1}$, and therefore screening is effective for harmonics with $q \ll q_s$ and much less so for $q \geq q_s$.

The effect of electrostatic screening is the result of a *density* response while the correlation energy is determined by the *filling factor* of quantum states. When the density and the filling factor are equal to each other, that is, when the relevant length scales are much larger than the extent of the wavefunction, electrostatic and correlation energies behave coherently. In regions with *linear screening*, the one-particle quantum states that correspond to the minimum total energy of the system, have energies equal to the chemical potential. Although electrostatic screening is limited by the effects of correlations, and therefore, the electrostatic potential differs significantly from the chemical potential, individual energies, being the combinations of electrostatic and correlation contributions, fall around the chemical potential ($\pm k_B T$). Such states are filled partially and behave like a metal.

We would like to point out that even in the regions of partial filling there is still a residual electric field due to the finite screening length. For disorder potentials with $q > q_s$ the *electrostatic interaction* becomes ineffective. The range of wavevectors q where, in turn, the *correlation potential* becomes unable to alter the individual particle energies prescribed by electrostatic potential is defined by the condition that there be one particle within one fluctuation of the potential. We come to the conclusion that both electrostatic interaction and correlation potential do not affect individual particle energies for external harmonic potentials with the wavelengths $\lambda \leq l$. Rephrasing it, if the system "knows" about the presence of fluctuations of potential with $\lambda \leq l$, the states in this system are either completely filled or empty; no partial filling is possible. This sets a natural limit to electrostatic approximation: if the *external* potential fluctuations with the wavelength $\lambda \leq l$ are important, the full quantum problem needs to be solved.

IV. FINITE TEMPERATURE EFFECTS. GENESIS OF THE QHE

We now consider how the picture presented in the previous Section manifests itself in the QHE. Our object of attention will be so-called clean systems which are experimentally realized in GaAs/AlGaAs heterojunctions[5]. In those systems, a random potential is mainly created by remote Silicon donors placed in a layer parallel to the 2DES, a spacer thickness d (of the order of several hundred Å) away from it. As demonstrated by Efros[5], a random distribution $C(\mathbf{r})$ of such donors produces an electrostatic potential $\Phi(\mathbf{r})$ in the plane of 2DES which is given by:

$$\Phi(\mathbf{r}) = 2\pi \frac{e^2}{\epsilon} \int \frac{d^2 q}{q} C(\mathbf{q}) e^{i\mathbf{q}\mathbf{r} - qd} , \qquad (10)$$

where

$$C(\mathbf{q}) = \int \frac{d^2 r}{(2\pi)^2} C(\mathbf{r}) e^{-i\mathbf{q}\mathbf{r}} . \qquad (11)$$

Equation (10) clearly shows that harmonics of the random potential with $qd > 1$ are exponentially suppressed. This lead the authors of Refs. 5,15,16 to the conclusion that short-wavelength potential fluctuations are not important. Let us analyze the picture of the QHE under this assumption.

For filling factors ν close to an integer, say, from the upper side, $\nu = N + \delta$, there are very few electrons above the top-most filled Landau level. As shown in Refs. 5,15, the transition between the wavelengths of the harmonics that are effectively screened (linear screening regime) and those which are poorly screened (non-linear screening regime) can be written as

$$R_c = \pi l^2 \frac{\sqrt{C}}{\delta} , \qquad (12)$$

where C is an average areal donor density. For sufficiently small δ, R_c becomes large and surpasses the spacer thickness d. In this case some strong harmonics in the random potential are left unscreened since screening would require many more electrons from the lower Landau levels to participate and, therefore, many excitations over the magnetic gap $\hbar\omega_c$. If the region with non-linear screening percolates, the QHE is observed since in the non-linearly screened regions only completely filled or empty Landau levels are present. For spin-degenerate Landau levels a rough estimate gives $|\delta| < \delta_c$, with $\delta_c \simeq 0.1$ for the width of the QH plateau under typical conditions. This clearly contradicts with the observed QH plateaus at small temperatures which are very wide.

Another weakness of the theory, originating in the neglect of the short wavelength harmonics of the disorder potential, is its prediction for the temperature dependence of the QH plateaus widths[5]. Contrary to the experimentally observed monotonous increase of the width with lowering the temperature[19] with a feature at a non-universal temperature of about $T \simeq 300$mK, this theory predicts narrowing of the plateaus with cooling down.

Based on the discussion following Eq. (9) we will show that it is possible to complement the present theory to build a consistent description of the QH effect spanning a wide range of temperatures. Realistic conditions for observing the QHE assume some finite temperature T. This, of course, affects the density of states. The sharp peak at Fermi energy is smeared into a broader line. Consider a filling factor at which the region of linear screening percolates, $|\delta| \geq \delta_c$. Then the states inside the percolating region correspond to states inside the DOS peak near the chemical potential μ, and their energies fall within the temperature T around μ. This description is correct as long as linear screening is in effect. We have argued that all harmonics *are* linearly screened (in the sense explained above) as long as the wavelength $\lambda \gg l$. In other words, even inside a linearly screened region, fluctuations of the disorder potential with sufficiently short
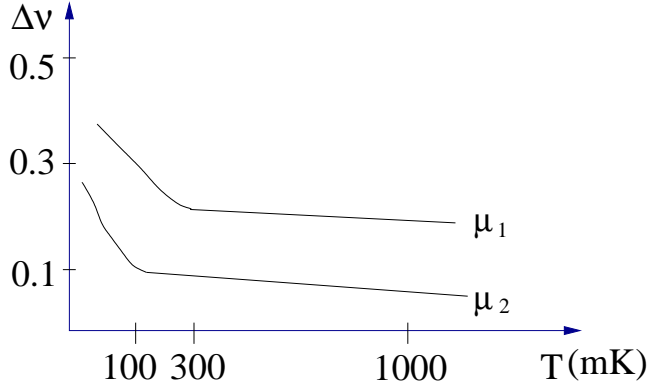
FIG. 3. Temperature dependence of the QH plateau width. Measurements for two samples with mobilities $\mu_1 < \mu_2$ are shown (after Gee et al.[19]).
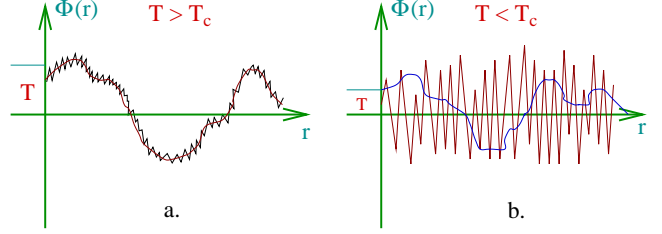

FIG. 4. Fluctuations of the self-consistent electrostatic potential in regions of linear screening. Long wavelength fluctuations are screened to a size of the order of the temperature. Short wavelength fluctuations are not screened. a.: $T > T_c$, short wavelength fluctuations are negligible compared with T; b.: $T < T_c$, temperature is small compared with the unscreened fluctuations.

wavelengths are not significantly affected by electron interactions. However, if the amplitude of these fluctuations is much smaller than the temperature they are not seen by the system; they appear like small ripples on top of big waves [Fig. 4(a)] and the occupation numbers of the states involved are practically unchanged.

When the size of these unscreened fluctuations becomes comparable with temperature, however, the distribution of occupation numbers changes dramatically, and eventually, when the amplitude of high frequency harmonics exceeds the temperature, the possibility of partial filling is eliminated. In this case the linearly-screened, metallic regions, break into pieces of completely filled or empty top-most Landau level. In each and all of these parts, the electrons behave as an incompressible liquid and depending on which filling factor percolates, the Hall conductance takes the corresponding quantized value [Ne^2/h or $(N-1)e^2/h$].

It is now possible to make a rough estimate for the crossover temperature. If we denote the crossover wavelength by λ_c and use Eq. (10), we find that the amplitude V_c of the harmonic λ_c is given by

$$V_c(q=2\pi/\lambda_c) = V_0 e^{-2\pi d/\lambda_c} , \qquad (13)$$

where V_0 is a typical amplitude of the long-wavelength fluctuations. Because of the uncertainty in λ_c and the exponential dependence of V_c on it, the evaluation of V_c can be unreliable. We note, however, that for $\lambda_c = d$, $V_c = 2 \times 10^{-3} V_0$. Since V_0 is usually of the order of several $\hbar\omega_c$, and for typical densities $\nu = 1$ corresponds to $\hbar\omega_c \simeq 100$K, we have an estimate of several hundred millikelvin for V_c. As discussed above, V_c has also a meaning of crossover temperature T_c, and we therefore conclude that as we lower the temperature around the value $T \sim T_c$ given by the right-hand side of Eq. (13), the system crosses over from dissipative to non-dissipative, quantum Hall regime. This cross-over temperature is clearly non-universal since it depends on the magnetic field, the spacer thickness and the strength of disorder.

We will show below that above the crossover temperature the width of the QH plateau is almost independent of the temperature with a slow growth tendency towards lower temperatures. It is clear that upon crossing T_c down the width of the plateau starts growing fast since more short-wavelength harmonics become "seen" by the system and larger areas of previously metallic regions break into incompressible pieces. Such a crossover feature has been seen in several experiments at temperatures below 1K[9,14,19]. At even lower temperatures the long-range part of disorder potential becomes completely irrelevant and one can treat the system as composed of independent electrons in a short-range disorder potential. It is believed that at zero temperature such a system exhibits sharp transitions between the QH plateaus[20] so that there is no percolating metallic region at any filling factor. For sufficiently narrow transitional region between the plateaus the behavior becomes universal, i.e. independent of the particular disorder configuration and electron density. This feature is entirely the consequence of the short-range nature of the random potential. Therefore, in order to approach the universal regime the temperature needs to be reduced at least below the crossover temperature. For very clean samples (for example, with a spacer thickness of around 1000 Å[19]) T_c can become less than 10mK and universal behavior is usually not achieved.

At this point, it is important to emphasize that, for the relatively clean samples under consideration, the QH plateaus are wide only at temperatures $T \stackrel{<}{\sim} T_c$, where the quantum effects mentioned above are dominant. In other words, for a wide range of magnetic fields, the QH effect is only observed in the low temperature regime, and as we increase the temperature above T_c, a metallic region percolates, extended states are close to the Fermi energy and dissipative processes become important.

Consider now the situation when the filling factor is outside the plateau at $T > T_c$. The extended state is then in the compressible region (within $E_F \pm k_B T$), and all states in the incompressible region are localized. Let us

now consider the situation in the low-temperature regime ($T < T_c$). Since states in the formerly incompressible region are not majorly affected by quantum effects, we must conclude that any extended state must come from one of the formerly partially filled states. It immediately follows that the distance between the energy of extended states and the Fermi level, or the width of the conventional gaussian DOS in this "quantum regime" cannot exceed $k_B T_c$.

There is one additional dramatic consequence of the constraints on the QH liquid in the "quantum regime": there is a maximum "wavelength" of a few magnetic lengths for an alternating structure of completely filled and completely empty states. Consider the transformation of the *compressible* liquid as the temperature is lowered past T_c. Only states in the compressible regions can vary the occupation numbers, but these variations are constrained by the necessity to keep Coulomb energy close to the minimum. In order to achieve this, the occupation numbers must alternate with a period comparable to the magnetic length, so that the charge density is not changed significantly. This is in a deep contrast with a conventional perception of the incompressible QH liquid as homogeneous on almost macroscopic length scales.

Similar arguments can be used to study the fractional QHE (FQHE). In general, one would have to talk in terms of composite fermions rather than electrons. This would complicate the picture: the Hamiltonian for composite fermions contains a term which makes the effective magnetic field dependent on the local charge density. However, in the incompressible liquid with a quantized filling factor (i.e. $\nu = 1/3$), the density is constant. In incompressible regions the disorder potential is unscreened and, therefore, the electric field in these regions is of the order of the bare electric field ($\simeq 0.1 \text{V}/\mu\text{m}$). Since the FQHE gap for the strongest fraction does not exceed 1meV, the width of the FQHE liquid may not exceed 2–3 l_B. This, most probably, makes it impossible to observe narrow electrostatic-type FQHE plateaus above the corresponding crossover temperature. It also creates additional requirements on a consistent theory of the FQHE. Since the FQHE is thought to be a collective effect, only the liquid with a certain degree of homogeneity can display the FQHE: the narrow rivers mentioned above do not satisfy this property. Only few electronic states across the river may simultaneously be the part of FQHE liquid. These speculations demonstrate the need for modifications in the Laughlin state to consistently describe the FQHE in a disordered system.

Different approach has to be used in "dirty" samples realized in InAs/AlInAs heterojunctions or in Si MOS-FETS. Due to the different origin of the disorder potential, the short-wavelength harmonics are not suppressed exponentially and universal behavior can, in general, be observed at any temperature where the system exhibits the QHE. If a QH plateau exists in such a system it is always wide, and temperature induced onset of dissipation occurs at temperatures which have little dependence on
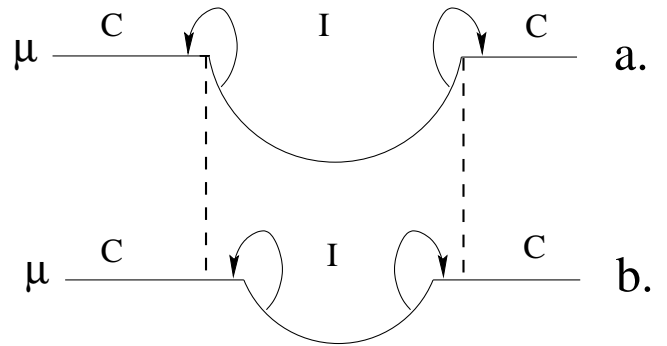

FIG. 5. Illustration of the melting mechanism of the QH system. I stands for "incompressible", C for "compressible". Thermal fluctuations increase the energies of states near the compressible regions.

the filling factor within the plateau.

A. Melting of the QHE liquid

This temperature dependent picture can be extended to the other limit of temperature $T \gg T_c$. Then only long-wavelength fluctuations of the potential are relevant in both linearly and non-linearly screened regions. Since the QHE is observed when the latter one percolates, one may ask a question of what are the possible mechanisms that destroy the non-dissipative regime as the temperature is increased. The amplitude of non-linearly screened harmonics is of the order of $\hbar\omega_c$ while the root mean square of the electric field is the same as for bare disorder potential $\mathcal{E} = 0.1\hbar\omega_c/el$. If excitations over the magnetic gap were a relevant mechanism for melting, the corresponding temperature T_m would be of the order of $\hbar\omega_c$ while the QHE disappears completely at a temperature an order of magnitude smaller. The following mechanism gives the right order of magnitude estimate for T_m.

Consider two isolated metallic lakes separated by an incompressible region (Fig. 5). Energies of the states in the lakes can be considered to be equal to the chemical potential μ; states inside the incompressible region are completely occupied and have energies lower than μ. Consider a state in the incompressible region nearest to one of the lakes [Fig. 5(a)]. Its energy relative to μ can be estimated as $\epsilon = -e\mathcal{E}l$, since the average extent of a state is of the order of the magnetic length l. If the temperature T is comparable to ϵ, the state becomes partially occupied and thermal fluctuations force its energy to raise closer to the chemical potential. This can happen only due to interactions with other electrons both in the lake and in the incompressible region. As a result of these interactions the size of the completely filled region shrinks and all states raise their energies as well. This situation repeats now with the new nearest-to-the-lake state [Fig. 5(b)].

It might happen that, during such a chain process, the electric field close to the boundary increases. Then the process is locked in this region unless the temperature is further increased. A natural estimate for the thermal breakdown temperature of the QHE is then

$$T_m = e\,\mathcal{E}\,l = 0.1\,\hbar\omega_c \ . \tag{14}$$

This formula gives a range from a few to several degrees Kelvin, in clean samples, in qualitative agreement with most experiments[8,9].

B. Plateau width dependence on the number of the filled Landau levels

It is intuitively clear that the width of the QH plateau should be a decreasing function of the number of the filled Landau levels. To understand such dependence we notice that it is the charge density (not the filling factors) which is responsible for screening of the external potential. Consider the same sample at different magnetic fields: in the first case the completely filled first LL percolates, in the second it is the filled second LL ($\hbar\omega_c^{(1)} > \hbar\omega_c^{(2)}$). The charge density distribution in the first case leads to the unscreened fluctuations of the potential of the order of $\hbar\omega_c^{(1)}$. However, such deep fluctuations are impossible in the second case: as soon as the potential falls below $\hbar\omega_c^{(2)}$, the 3rd LL starts filling. Therefore, the regions with partially filled 3rd LL at smaller magnetic fields are larger in size than the corresponding compressible regions with partially filled 2nd LL at stronger magnetic fields. Additional electrons on the top 3rd LL must have come from some other regions in the system: overall it should be neutral on sufficiently large length scales. The only possible reservoir for such electrons are the percolating incompressible regions. By reducing the density, these incompressible regions decrease in size and part of them become metallic. We conclude that both the regions with partially filled 2nd and 3rd LL's are larger than their counterparts (1st and 2nd LL's respectively) at stronger magnetic fields. Therefore, the QH plateau becomes narrower with decreasing the magnetic field, as expected.

C. Density of states as a function of temperature

We now return to the DOS and seek to understand how it changes when the temperature is varied. At high temperatures, $T \gg T_c$, there is a peak around the Fermi energy with a width of the order of temperature (Fig. 2). The amplitude of any unscreened harmonic in the linear screened regions is negligibly small, as compared to the temperature and cannot be seen inside the resonance curve. The tail in the DOS corresponds to states in the incompressible regions.

For temperatures $T_c \ll T \ll T_m$ this DOS is practically temperature independent (except for a rounding of the peak at higher temperatures). According to the previous discussion, as T approaches and crosses below T_m, more states from the tail of the DOS are pumped into the peak. The peak width, of the order of the temperature, changes very little compared to the total width of the DOS. Therefore, in this range of temperatures, the DOS changes by increasing the height of the peak at the expense of the reduction of the weight of the tail.
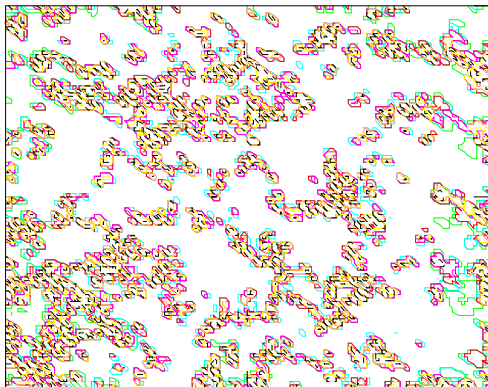
On the other side, for $T \ll T_c$, the amplitude of unscreened fluctuations becomes greater than the width the peak would have at this temperature in the absence of short-range disorder. The width of the peak in the DOS in this case is determined by the strength of short-wavelength disorder potential and is independent of temperature. This is a common non-interacting symmetric DOS with a few modifications: the existence of a residual DOS at the tails spreading out to energies comparable with $\hbar\omega_c$, and the distance between the line of symmetry of the DOS and the Fermi level is of the order of $T_c \ll \hbar\omega_c$. The Fermi energy is either below or above the center of the peak depending on the filling factor.

We were able to simulate numerically the state of the system for $T > T_c$. The procedure consists of minimization of the free energy with respect to local density at finite temperature. The free energy of the system can be written as
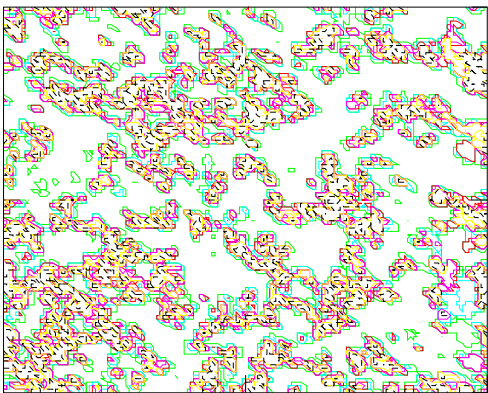
$$\mathcal{F} = \mathcal{H}_1 + k_B T \int d^2r \sum_N \Big(n^{(N)}(\mathbf{r}) \log[n^{(N)}(\mathbf{r})] +$$
$$+ [1 - n^{(N)}(\mathbf{r})] \log[1 - n^{(N)}(\mathbf{r})] \Big) \ , \tag{15}$$

where $\mathcal{H}_1$ is defined in Eq. (3). The last term in Eq. (15) is the entropy of the system described by Fermi statistics. Fig. 6 demonstrates the density distribution in clean system with average occupation number around $\nu = 1.92$. Part (a) corresponds to a temperature $T = 1K < T_m$. Figure 6(b) corresponds to a slightly higher temperature, $T = 2K$: some expansion of the compressible region is observed. Finally, Fig. 6(c) represents the same system at $T = 3K$ which is apparently greater than T_m. We could bracket T_m to within an interval between 2.5K and 2.8K. We also observed very weak dependence of T_m on the filling factor. An interesting result produced by these calculations is that when the melting occurs at $\nu \geq 2.0$, a metallic percolating region is formed both on the first and second spin-degenerate Landau levels practically at the same time. All the features discussed above are well seen on these plots. Unfortunately, the electrostatic approximation by its nature does not allow us to probe the system at temperatures around and below T_c. A more sophisticated approach is needed to quantify and, probably, numerically illustrate the crossover picture. At the moment we are unable to present a satisfactory, quantitative model for the microscopical nature of the crossover regime as well as microscopic picture of inhomogeneous QH liquid at temperatures below T_c (see Refs. 21–23).
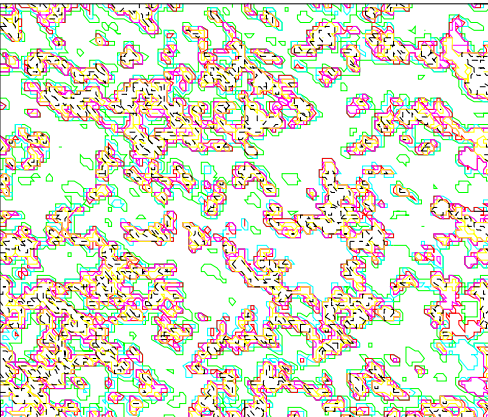
T = 1K, ν = 0.96

T = 2K, ν = 0.96

T = 3K, ν = 0.96

FIG. 6. Numerical simulations of the melting of the QH system. The same sample, with a filling factor $\nu = 1.92$, is studied at different temperatures. Shaded areas are compressible, white areas are incompressible. a. $T = 1$K; b. $T = 2$K; c. $T = 3$K. As the temperature is raised compressible areas finally percolate, destroying the QH regime.

V. DISSIPATIVE PROCESSES AND ENERGY RELAXATION IN THE QH REGIME

The characteristics of the DOS at temperatures above T_c has various implications on the dissipative dynamics in QH systems. We will concentrate on one of them, namely, the energy relaxation rate in the QH regime. By that we imply that we consider a "clean" system at $T > T_c$ and at such filling factors that the Hall conductance is quantized with high accuracy. In this Section we intentionally avoid calling this regime *non-dissipative* since it is dissipation, however small it might be, that we are interested in.

Assume that current is injected into the system. Since dissipative conductivity is finite, there is finite Joule heat released into the electronic system. Depending on the rate of relaxation of this energy from the electrons to the lattice, the 2DES will either be in quasi-equilibrium ($T_e \simeq T_l$), or the electron temperature will raise ($T_e > T_l$) to enhance this relaxation and achieve a steady state.

The problem of overheating of the 2DES in such systems has been addressed before in the study of different phenomena. Chow *et al.*[24] considered this effect in the *transitional* regime between QH plateaus, and argued that the effective electronic temperature dependence on the current contains needed information to reproduce one of the scaling exponents in the universal regime. Although the temperatures studied in Ref. 24 are apparently low (between 100mK and 500mK) they may still be higher than T_c since the sample they considered had a thick spacer ($d = 1800$Å). The authors argued that the energy relaxation rate to acoustic (piezoelectric) phonon modes scales, at these low temperatures and strong magnetic fields, as T_e^4; it was implicitly assumed that the energy relaxation within the 2DES due to inter-electron interaction was sufficiently fast and (with some justification) that diagonal conductivity σ_{xx} was independent of wavevector q and electron temperature T_e. The latter assumption obviously fails in the QH regime since finite conductivity results from either thermal activation ($\sim \exp[\epsilon_a/k_B T_e]$) or variable range hopping ($\sim \exp[-A/T_e^\beta]$), where ϵ_a is an activation energy, and β is some model-dependent positive power.

Our analysis of heating at filling factors within the QH plateaus is reminiscent of the analysis of Ref. 24. However, we will use explicitly the properties of the DOS, since it is unclear how to express the surface acoustic wave absorption probability through a global conductivity in the case of spatially inhomogeneous system (see also Ref. 25).

The notion of overheating of the electron system assumes that the electrons are considered to be in thermal equilibrium at some temperature T_e. This implies that all the energy pumped into the system is distributed throughout it at rate τ_e^{-1} which is much greater than the rate of any other energy relaxation process in the system. Important processes which are to be considered are the supply of energy from some external source and release

9

of this energy to a thermal bath. If τ_e is the shortest time scale in the problem, and if the temperature of the thermal bath is denoted as T_b, the 2DES temperature T_e is defined by the energy balance equation

$$P_{in} = P_{\text{out}}(T_e) , \qquad (16)$$

where P_{in} is the input power, and P_{out} is the energy relaxation rate from the electrons to the heat bath. The relevant question is how much the electron temperature needs to be raised so that Eq. 16 is satisfied. If energy relaxation to the lattice is slow, it is clear that considerable overheating ($T_e \gg T_b$) will be observed. Conversely, a fast relaxation mechanism assures that both temperatures are similar ($T_e \simeq T_b$).

The energy fed into the 2DES QH system is Joule heat associated with the injected current j and dissipative resistivity ρ_{xx}: $P_{in} = \rho_{xx} j^2$. This assumes that most dissipative processes do not involve electron-phonon interactions, otherwise part of the heating is directly applied to the lattice. At this point it is important to emphasize that in the QH regime ρ_{xx} is exponentially small. For the phonon emission power we write:

$$P_{\text{out}}(T_e) = 2\pi \sum_{i,f} \sum_{\mathbf{q}} \hbar\omega(\mathbf{q}) \, |M_{i,f}(\mathbf{q})|^2 \, f_i \, (1-f_f) \times$$

$$[\, (n_{\mathbf{q}}+1)\, \delta[\epsilon_i - \epsilon_f - \hbar\omega(\mathbf{q})] - n_{\mathbf{q}}\, \delta[\epsilon_i - \epsilon_f + \hbar\omega(\mathbf{q})]]\, . \quad (17)$$

where indices i and f correspond to initial and final electronic states respectively, $\mathbf{q}$ is the phonon momentum; $f_{i,f}(T_e)$ are fermionic filling factors at temperature T_e, $n_{\mathbf{q}}(T_b)$ is a bosonic filling factor at lattice temperature T_b, and $M_{i,f}(\mathbf{q})$ denotes the matrix element of the electron-phonon interaction. In Eq. (17) the first term inside the vertical brackets stands for phonon emission while the second corresponds to phonon absorption. To estimate P_{out} we make the following approximation: each electronic state can be visualized as a strip with a transverse size of the order of the magnetic length l stretched along some equipotential. Inside any individual potential fluctuation this potential can be linearized. The wave function in this linearized region can then be written as a solution of Landau equation in a homogeneous electric field. Solutions of this equation are enumerated by the local set of one-dimensional wave vectors k that have meaning of the position of the wave functions center[26], which for the lowest Landau level read

$$\psi_k(x,y) = \frac{1}{\pi^{1/4}\, l^{1/2}} exp\left[ikx - \frac{(y-y_0)^2}{2l^2} \right] , \qquad (18)$$

where x is the direction along the trajectory and $y_0 \simeq kl^2$ is the guiding center position. Energies of these states follow the external potential in the fluctuation:

$$\epsilon_k = \frac{1}{2}\hbar\omega_c + kl^2 e\mathcal{E} , \qquad (19)$$

with $\mathcal{E}$ being the typical electric field inside the fluctuation. Since the characteristic wavelength of potential

fluctuations in the regime we are studying is much larger than l, and any transition due to electron-phonon interaction is possible as long as initial and final states have substantial overlap, such transition occurs when the electron moves from one strip to another approximately l away in the same potential fluctuation. Using the dispersion relation for longitudinal acoustic phonons $\omega = c_L q$, with $q = [q_x^2 + q_y^2 + q_z^2]^{1/2}$, an estimate of $\mathcal{E} \simeq 0.1\, \hbar\omega_c/el$, and given that $c_L \sim 10^3$ m/s, one can make sure that single phonon processes are indeed allowed: $e\mathcal{E}\triangle y_0 = c_L q > c_L q_x \sim c_L \triangle y_0/l^2$ leads to $\mathcal{E} > 0.005\, \hbar\omega_c/el$, which is certainly the case. The matrix element $M_{i,f}(\mathbf{q})$ calculated with the wave functions from Eq. (18) depends on the wave vector of initial state only through a phase factor that is irrelevant in this problem.

Since each strip passes through many potential fluctuations (provided that the corresponding equipotential is sufficiently long) the electron on the strip probes many random configurations of the potential and, therefore, averages over the whole ensemble. Hence, one can use the ensemble averaged density of states $D(\epsilon)$ to describe the probability of the electron having particular initial and final energies, at least for electrons in the incompressible regions. In this approximation, Eq. (17) takes the form:

$$P_{\text{out}} = 2\pi \sum_{\mathbf{q}} |M(\mathbf{q})|^2 \, \frac{1}{\alpha - 1} \times \qquad (20)$$

$$\sum_{\epsilon_{\mathbf{k}}} \left[\alpha x\, \frac{D(\epsilon_k) D(\epsilon_k - c_L q)}{(\beta + x)(1+x)} - \beta x\, \frac{D(\epsilon_k) D(\epsilon_k + c_L q)}{(\beta x + 1)(1+x)} \right] ,$$

where we denoted $x = \exp[\epsilon_k/kT_e]$, $\alpha = \exp[c_L q/kT_b]$ and $\beta = \exp[c_L q/kT_e]$. Changing to an integral over ϵ_k and after simple manipulations we obtain:

$$P_{\text{out}} = k_B T_e \sum_{\mathbf{q}} |\tilde{M}(\mathbf{q})|^2 \, \frac{\alpha - \beta}{\alpha - 1} \times$$

$$\times \int_0^\infty dx\, \frac{D(k_B T_e \log x) D(k_B T_e \log x/\beta)}{(\beta + x)(1+x)} , \quad (21)$$

where we absorbed all numerical factors into a redefinition of the matrix element $\tilde{M}(\mathbf{q})$. We did not have to specify the exact form of electron-phonon interaction; it can, in fact, be of either piezoelectric or deformation potential nature. The relative contribution of these two types of interaction depends on the temperature[27], however both are of the same order of magnitude at temperatures around 1K. After averaging over phonon polarization, one gets matrix elements of the order of unity. We chose not to go into details of the form of these matrix elements (see Refs. 27–30) since for our purposes only the order of magnitude matters. We kept the factor $k_B T_e$ as it appeared with the change of integration variable for the sake of dimensionality.

The most important part defining the magnitude of P_{out} in Eq. (21) is the density of states $D(\epsilon)$. For example, if only states well below the Fermi level, with energies $\epsilon \ll k_B T_e$, have weight in the DOS then the range

of integration over x in Eq. (21) becomes exponentially narrow: the upper limit of integration is then of the order of $\exp[-\epsilon_0/k_B T_e]$, where ϵ_0 is the highest energy such that $D(\epsilon_0)$ is not exceedingly small. On the other hand, if the DOS spreads all the way to the Fermi level, the corresponding integral in Eq. (21) is not small nor is the emitted power P_{out}.

To see that energy relaxation rate due to phonon emission is sufficient to keep the difference $T_e - T_b$ negligibly small, we recall that Joule heating rate is suppressed exponentially by a factor of $\exp[-\epsilon_a/kT_e]$: dissipative transport is due to the electrons (or holes) in the extended states. Different mechanisms of scattering are available for these electrons. Scattering off the electrons in metallic, compressible regions is one mechanism leading to thermal equilibration within the 2DES. The other mechanism proposed in[10,11] involves interaction with lattice phonons. It is not clear which of these contributions to dissipative resistivity prevails. If under certain conditions the latter one dominates, the 2DES has to be in thermal equilibrium with the lattice and $T_e \simeq T_b$. Electron-electron scattering is the only way of releasing energy into 2DES. It is clear, however, from Eq. (21), as well as from intuitive arguments, that electrons in metallic regions are interacting with the phonons very effectively and are never a bottleneck for thermal equilibration with the lattice. In general, the metallic regions are always in thermal quasi-equilibrium with the lattice. They may not be equilibrated with the electrons in incompressible regions: in this case the QH system has to sustain the temperature gradients. However, it assumes that the dissipation of energy by incompressible electrons carrying the Hall current occurs via interaction with phonons: in the QH system diffusion (and therefore dissipative conductivity) occurs through inelastic scattering due to the non-degenerate nature of the states in long-range disorder potential.

Expression (21) is applicable in the transitional regime as well. However, when the metallic region percolates, dissipation is also greatly enhanced. Both Joule heating and phonon emission are determined in this case by the same DOS, that is by a finite fraction of the total number of states at the Fermi level. Observation of overheating in this regime[24] is therefore not surprising.

VI. CONCLUSIONS

In this paper we demonstrate that screening of long-range fluctuations of the disorder potential leads to a novel form of the density of states. Contrary to the conventional DOS for non-interacting electrons, this screened DOS possesses a distinctive peak at the Fermi level originating from states in metallic (compressible) regions. We argue that this form of DOS remains almost intact with changes in the magnetic field, the only variation being in the width of the peak and in the position of the energy of the extended states relative to the Fermi level. We also argue that, as the temperature of the system crosses below a certain value T_c, short-range fluctuations of the disorder potential become dominant and the nature of the states within the narrow peak in DOS changes. This T_c is the lower boundary of the range of validity of the electrostatic approximation.

For filling factors that only exhibit the QHE at low temperatures ($T \overset{<}{\sim} T_c$), short-range fluctuations dominate and the full quantum problem needs to be solved. Based on numerical and analytical results obtained by a number of authors, one can expect that for non-interacting electrons, the DOS has symmetric gaussian shape, centered at the energy of extended states. In light of our developments, the assumption of non-interacting electrons needs to be amended by the requirement that the charge density remain unchanged when the temperature is lowered through T_c, that is, it is still prescribed by electrostatic solution even at low temperatures. This leads not only to a constraint on the distance between the energy of extended states and the Fermi level never to exceed T_c but also to a more dramatic conclusion: namely, that in these, short-range fluctuations dominated regime, the percolating quantum Hall liquid can nowhere be wider than few magnetic lengths. This conclusion calls for a closer look at the existing quantum theory of the QHE which is based on the assumption of an homogeneous system.

The current common perception of the QHE is that it is an entirely quantum phenomenon. In this sense, the theory of the QHE refers to the lower temperatures ($T < T_c$) and to the magnetic fields away from the center of the plateaus. Yet it is important to bear in mind that this QH system is very inhomogeneous and that the "pure" considerations do not apply to the whole system but rather to the regions where the disorder potential is metallically screened. All the observed quantum effects, including the universal behavior at low temperatures, have their origin in these regions. The complete theory of the IQHE has to account for this non-trivial microscopic structure of the 2DES.

Based on the understanding of the QH system described by the electrostatic approximation, we also present a model of melting of QH liquid at higher temperatures. We were able to explain why the electrostatic solution predicts narrow quantum Hall plateaus while experimentally observed plateaus are usually much wider, this difference originating in the crossover behavior described above.

Finally, we applied the derived form of the DOS to the study of dissipative processes in QH system. We showed that since Joule heating is limited in the QH effect by a very low dissipative conductivity, and the relaxation of energy due to the coupling to phonons is very effective (because of the large number of states on the Fermi level), electronic overheating cannot occur (or is negligible) in the QH regime.

[1] T. Ando, A. B. Fowler and F. Stern, Rev. Mod. Phys. **54**, 437 (1982); T. Ando and H. Aoki, J. Phys. Soc. Japan **54**, 2238 (1985).

[2] F. Wegner, Z. Physik B **51**, 279 (1983).

[3] B. Huckestein and B. Kramer, in *High Magnetic Fields in Semiconductor Physics III, Quantum Hall Effect, Transport and Optics*, Springer Series in Solid-State Sciences No. 101, edited by G. Landwehr (Springer, Berlin), p. 70, (1992).

[4] R. E. Prange in *The Quantum Hall Effect*, edited by R. E. Prange and S. M. Girvin (Springer-Verlag, 1987).

[5] A. L. Efros, Sol. St. Comm. **65**, 1281 (1988); *ibid.* **67**, 1019 (1988).

[6] D. B. Chklovskii, B. I. Shklovskii, and L. I. Glazman, Phys. Rev. B **46**, 4026 (1992).

[7] V. Tsemekhman, K. Tsemekhman, C. Wexler, J.H. Han, and D. J. Thouless, Rev. B, **55**, R10201, (1997)

[8] G. Ebert, K. von Klitzing, K. Ploog and G. Weimann, J. Phys. C **16**, 5441 (1983).

[9] S. Komiyama, T. Takamasu, S. Hiyamizu and S. Sasa, Solid State Commun. **54**, 479 (1985).

[10] D. G. Polyakov and B. I. Shklovskii, Phys. Rev. Lett. **70**, 3796 (1993).

[11] D. G. Polyakov and B. I. Shklovskii, Phys. Rev. B **48**, 11167 (1993).

[12] D.-H. Lee, S. Kivelson and S.-C. Zhang, Phys. Rev. Lett. **68**, 2386 (1992).

[13] A. V. Gurewich and R. G. Mints, Pis'ma Zh. Eksp. Teor. Fiz. **39**, 318 (1984) [JETP Lett. **39**, 446 (1984).

[14] P. Svoboda, G. Nachtwei, C. Breitlow, S. Heide, M. Cukr, Semiconductor Science and Technology **12**, 264 (1997).

[15] A. L. Efros, F. G. Pikus, V. G. Burnett, Phys. Rev. B **47**, 2233 (1993).

[16] F. G. Pikus, A. L. Efros, Phys. Rev. B **47**, 16395 (1993).

[17] L. D. Landau and E. M. Lifshitz, *Quantum Mechanics* (Pergamon Press, Oxford, New York) (1977).

[18] G. Fano and F. Ortolani, Phys. Rev. B **37**, 8179 (1988).

[19] P. J. Gee *et al*, in *Conference Workbook, 11th International Conference "High Magnetic Fields in Semiconductor Physics"*, p. 392, (1994).

[20] B. Huckestein, Rev. Mod. Phys **67**, 357 (1995).

[21] L. Pryadko, Private Communication; L. P. Pryadko, K. Chaltikian, Phys. Rev. Lett. **80**, 584 (1998).

[22] A. A. Koulakov, M. M. Fogler, B. I. Shklovskii, Phys. Rev. Lett., **76**, 499, (1996).

[23] E. Shimshoni, A. Auerbach, A. Kapitulnik, Phys. Rev. Lett. **80**, 3352 (1998); E. Shimshoni, A. Auerbach, Philosophical Magazine B **77**, 1107 (1998).

[24] E. Chow, H. P. Wei, S. M. Girvin, M. Shayegan, Phys. Rev. Lett. **77**, 1143 (1996).

[25] A. L. Efros, Y. M. Galperin, Phys. Rev. Lett. **64**, 1959 (1990).

[26] H. L. Zhao, S. Feng Phys. Rev. Lett. **70**, 4134 (1993).

[27] P. J. Price, J. Appl. Phys. **53**, 6863 (1982).

[28] G. A. Toombs, F. W. Sheard, D. Neilson and L. J. Challis, Solid State Commun. **64**, 577 (1987).

[29] F. Dietzel, W. Dietsche and K. Ploog, Phys. Rev. B **48**, 4713 (1993).

[30] S. Tamura and H. Kitagawa, Phys. Rev. B **40**, 8485 (1989).